

2.10 Roots of quadratic and cubic equations

Quadratic equations

Equation	$ax^2 + bx + c = 0 \quad (a \neq 0)$	(2.454)	x variable a, b, c real constants
Solutions	$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	(2.455)	x_1, x_2 quadratic roots
	$= \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$	(2.456)	
Solution	$x_1 + x_2 = -b/a$	(2.457)	
combinations	$x_1 x_2 = c/a$	(2.458)	

Cubic equations

Equation	$ax^3+bx^2+cx+d=0 \quad (a \neq 0)$	(2.459)	x variable a,b,c,d real constants	
Intermediate definitions	$p=\frac{1}{3}\left(\frac{3c}{a}-\frac{b^2}{a^2}\right)$	(2.460)	D discriminant	
	$q=\frac{1}{27}\left(\frac{2b^3}{a^3}-\frac{9bc}{a^2}+\frac{27d}{a}\right)$	(2.461)		
	$D=\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2$	(2.462)		
If $D \geq 0$, also define:		If $D < 0$, also define:		
$u=\left(\frac{-q}{2}+D^{1/2}\right)^{1/3}$		(2.463)	$\phi=\arccos\left[\frac{-q}{2}\left(\frac{ p }{3}\right)^{-3/2}\right]$	(2.467)
$v=\left(\frac{-q}{2}-D^{1/2}\right)^{1/3}$		(2.464)	$y_1=2\left(\frac{ p }{3}\right)^{1/2}\cos\frac{\phi}{3}$	
$y_1=u+v$		(2.465)		
$y_{2,3}=\frac{-(u+v)}{2}\pm\mathbf{i}\frac{u-v}{2}3^{1/2}$		(2.466)	$y_{2,3}=-2\left(\frac{ p }{3}\right)^{1/2}\cos\frac{\phi\pm\pi}{3}$	
1 real, 2 complex roots (if $D=0$: 3 real roots, at least 2 equal)		3 distinct real roots		
Solutions ^a	$x_n=y_n-\frac{b}{3a}$	(2.470)	x_n	cubic roots ($n=1,2,3$)
Solution combinations	$x_1+x_2+x_3=-b/a$	(2.471)		
	$x_1x_2+x_1x_3+x_2x_3=c/a$	(2.472)		
	$x_1x_2x_3=-d/a$	(2.473)		

^a y_n are solutions to the reduced equation $y^3 + py + q = 0$.

